1 Evaluate: $\int sin^3(x)dx$:

First break apart the $sin^3(x)$ into $sin^2(x)sin(x)$. Then use the identity $sin^2(x) = 1 - cos^2(x)$ to get:

$$\int \left(1 - cos^2(x)\right) sin(x)dx$$

Using a u-substitution where $u = cos(x)$ and $du = -sin(x)dx$ the integral becomes:

$$-\int \left(1 - u^2\right) du = -\left(u - \frac{1}{3}u^3\right) + c = -u + \frac{1}{3}u^3 + c$$

Subbing back in for u we get:

$$\frac{1}{3}cos^3(x) - cos(x) + c$$

2 Evaluate: $\int x^5 e^{x^3} dx$:

Rewrite the equation:

$$\int x^2 x^3 e^{x^3} dx$$

Use a u, du substitution (with the variable t):

$$t = x^3$$

$$dt = 3x^2 dx$$

Thus:

$$x = \sqrt[3]{t}$$

Thus the integral becomes:

$$\frac{1}{3}\int (\sqrt[3]{t})^3 e^t dt = \frac{1}{3}\int te^t dt$$

Using integration by parts ($uv - \int vdu$):

$$u = t \qquad dv = e^u du$$

$$du = dt \qquad v = e^u$$

Thus the integral becomes:

$$\frac{1}{3}(te^t - \int e^t dt) = \frac{1}{3}(te^t - e^t) + c$$

Subbing back into the the original $t = x^3$ substitution, we have:

$$\int x^5 e^{x^3} dx = \frac{1}{3}(x^3 e^{x^3} - e^{x^3}) + c$$

or:

$$\int x^5 e^{x^3} dx = \frac{1}{3}x^3 e^{x^3} - \frac{1}{3}e^{x^3} + c$$

3 Evaluate: $\int \frac{1}{x^2\sqrt{x^2-9}}dx$

Use the substitution $x = 3sec(\theta)$ and $dx = 3sec(\theta)tan(\theta)d\theta$ Thus:

$$\sqrt{x^2-9} = \sqrt{(3sec(\theta))^2-9} = \sqrt{9sec^2(\theta)-9} = \sqrt{9(sec^2(\theta)-1)} = 3\sqrt{tan^2(\theta)} = 3tan(\theta)$$

So the integral becomes:

$$\int \frac{1}{(3sec(\theta))^2 3tan(\theta)} 3sec(\theta)tan(\theta)d\theta = \frac{1}{9}\int \frac{1}{sec(\theta))}dx = \frac{1}{9}\int cos(\theta)dx = \frac{1}{9}sin(\theta) + c$$

Solving $x = 3sec(\theta)$ we can create a triangle that has an opposite side of $\sqrt{x^2-9}$ and hypotonose of x. Thus the integral becomes:

$$\int \frac{1}{x^2\sqrt{x^2-9}}dx = \frac{1}{9}\frac{\sqrt{x^2-9}}{x} + c$$

4 Evaluate: $\int \frac{x}{\sqrt{1+2x}}dx$

There were two different method to compute this derivative:

4.1 Method 1: Integration by parts

Use the substitution $u = x$ and $dv = (1+2x)^{-\frac{1}{2}}dx$, then $du = dx$ and $v = \int(1+2x)^{-\frac{1}{2}}dx = (1+2x)^{\frac{1}{2}}$. Therefore $uv - \int vdu$ becomes:

$$x(1+2x)^{\frac{1}{2}} - \int (1+2x)^{\frac{1}{2}}dx$$

To solve $\int(1+2x)^{\frac{1}{2}}dx$, we use an s substitution, where $s = 1+2x$ and $ds = 2dx$. This gives that $dx = \frac{1}{2}dx$:

$$\int (1+2x)^{\frac{1}{2}}dx = \frac{1}{2}\int (s)^{\frac{1}{2}}ds = \frac{1}{2}s^{\frac{3}{2}}\frac{2}{3} = \frac{1}{3}s^{\frac{3}{2}}$$

Subbing in for s ($s = 1+2x$) we get:

$$\int \frac{x}{\sqrt{1+2x}}dx = x(1+2x)^{\frac{1}{2}} - \frac{1}{3}(1+2x)^{\frac{3}{2}} + c$$

4.2 Method 2: U-substitution

Use the substitution $u = 1+2x$ and then $du = 2dx$. Thus $dx = \frac{1}{2}du$ and $x = \frac{u-1}{2}$.

$$\int \frac{x}{\sqrt{1+2x}}dx = \frac{1}{2}\int \frac{\frac{u-1}{2}}{u^{\frac{1}{2}}}du = \frac{1}{4}\int u^{\frac{1}{2}} - u^{-\frac{1}{2}}du = \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) + c.$$

Subing in for $u = 1+2x$ we get the final result of:

$$\frac{1}{6}(1+2x)^{\frac{3}{2}} - \frac{1}{2}(1+2x)^{\frac{1}{2}} + c.$$

The results from method 1 and method 2 produce different results, but if you check them, by differentiating them, they are equivelent.