Exam 3: solutions. Please see Greg or Prof. Silver for answers to the questions that are not listed below.

Multiple choice problems (there was some variation among the versions)

---- Find a set of polar coordinates for the following rectangular coordinates: (-1,1)

Remember to STAY in Quadrant II: use $\tan\theta = y/x$ *and* $r^2=x^2 + y^2$ *Ans:* $(\sqrt{2}, \frac{3\pi}{4})$

---- Which integral represents the area of the figure represented by r = 4sinθ from [0, π]?

Formula for area is: $\frac{1}{2}\int_a^b [f(\theta)]^2 \, d\theta$ therefore: $\frac{1}{2}\int_0^{\pi} [4\sin\theta]^2 \, d\theta = \frac{1}{2}\int_0^{\pi} [16\sin^2\theta \, d\theta = \int_0^{\pi} 8\sin^2\theta \, d\theta$ (answer)

Numeric response problems:

----How many petals (or loops) does the graph of the rose described by r = 2cos(4θ) have?

Since four is an even number, we double it: 4 times 2 = 8 petals (answer).

----A constant force with vector representation **F** = 6**i** + 10**j** + 7**k** moves an object along a straight line from the point (4, 5, 3) to the point (3, 7, 10). Find the work done.

Force vector: <6, 10, 7> and displacement vector: <-1, 2, 7>. Use dot product. Ans: 63

----Find the scalar projection of **b** onto **a**. **a** = <8, 6>, **b** = <7, 10>

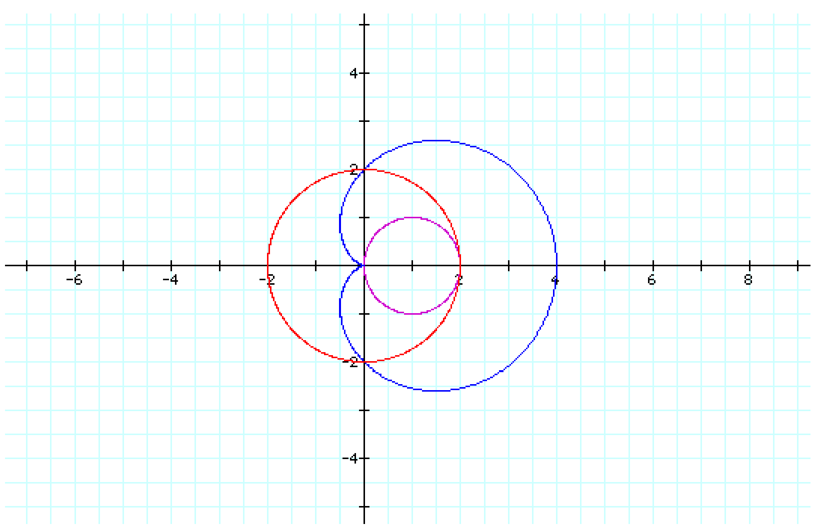
Use formula: $\frac{a \bullet b}{\| a \|}$ *ans: 11.6*

----Match each equation with the COLOR of the graph (see transparency)

_______small circle (purple)___ r = 2cosθ

_______cardioid (blue)_____ r = 2 + 2cosθ

_______large circle (red)____ r = 2



Partial credit problems:

--Find the focus of the parabola given by the equation

$$-2y = x^2 + 2x - 1$$

$-2y + 1 = x^2 + 2x + ?$ *you need to complete the square on the right*

$-2y + 1 + 1 = x^2 + 2x + 1$ *(add one to both sides)*

$-2y + 2 = (x+1)^2$

$-2(y-1) = (x+1)^2$ *therefore:* $(h,k) = (-1,1)$ *and* $4p = -2$; *therefore* $p = -\frac{1}{2}$ *and focus* : $(h, k+p) = (-1, \frac{1}{2})$

----***Use your knowledge of vectors*** to decide whether the triangle with vertices A(-1,1,3), B(3,-1,1) and C(1,3,-1) is an equilateral triangle, an isosceles triangle, or just a general triangle.

One method: find the length of the sides of triangle:

$\overrightarrow{AB} = < 3-(-1),\ -1-1,\ 1-3 > = < 4, -2, -2 >$

$\overrightarrow{AC} = < 1-(-1),\ 3-1,\ -1-3 > = < 2, 2, -4 >$

$\overrightarrow{BC} = < 1-3, 3-(-1), -1-1 > = < -2, 4, -2 >$

$\| \overrightarrow{AB} \| = \sqrt{16+4+4} = \sqrt{24}$; $\| \overrightarrow{AC} \| = \sqrt{4+4+16} = \sqrt{24}$; $\| \overrightarrow{BC} \| = \sqrt{4+16+4} = \sqrt{24}$

therefore the triangle is an *equilateral* triangle, since the 3 sides have equal lengths.

----Your younger sister recently learned in her science class that Kepler's First Law says: "The orbit of every planet is an ellipse with the sun at one of the foci." She knows that this law can also be applied to the moon's orbit around the earth.

She is now in the process of creating a model that represents the elliptical path of the moon as it orbits the earth. In her model, this path has vertices of (±4,0), and is centered at the origin. The minor axis has a length of 6 units.

a) Write an equation that represents the elliptical path of the moon in your sister's model.

Equation for an ellipse is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since the ellipse is centered at the origin, we have (h,k) = (0,0)

Since the vertices are (±4,0)*, this means that the value for "a" is 4*

Since the minor axis has a length of 6, this means that the value for "b" is 3.

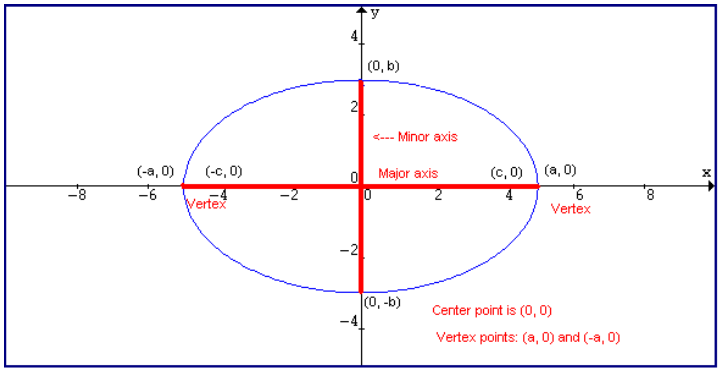
Therefore, the equation for this ellipse is: $\frac{x^2}{16} + \frac{y^2}{9} = 1$

b) In this model, what could be the coordinates for the center of the earth?

The earth's position is at one of the foci. The foci lie on the major axis, c units from the center, and: $c^2 = a^2 - c^2$. *It follows that* $c^2 = 16 - 9 = 7$ *and* $c = \pm\sqrt{7}$

Therefore the earth's position can be at $(-\sqrt{7}, 0)$ *or* $(\sqrt{7}, 0)$

c) Make a sketch of the graph that represents your sister's model (path of the moon, position of the earth). *Here, $a = \pm 4$, $b = \pm 3$, and $c = \pm\sqrt{7}$. The earth can be positioned at either foci. The moon is traveling around the ellipse.*



----For what values of **a** are the vectors $< -33, a, -3>$ and $< a, a^2, a>$ orthogonal? *(numbers varied in this problem, but the method was the same: Use the dot product: If two vectors are orthogonal (i.e., form a right angle) then $u \bullet v = 0$*

$-33a + a^3 - 3a = 0$ ... now combine like terms and rewrite in descending order: $a^3 - 33a - 3a = 0$ ... now factor out an "a": $a(a^2 - 36) = 0$ ... factor some more: $a(a + 6)(a - 6) = 0$.

So, the vectors will be orthogonal when $a = 0$, $a = 6$ and $a = -6$ (note: when a is zero, then one vector is the zero vector, but the zero vector is orthogonal to all other vectors – see page 809 in text.)